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Quantifying the contribution of a subpopulation to inequality

An application to Mozambique

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Abstract: In this paper, I quantify the contribution of a subpopulation to inequality. This is defined as the sum of the contributions of its members, with these contributions computed as the impact on inequality of a small increase in the population mass at each point of the distribution (using the Recentered Influence Function). The decomposition is shown to verify various attractive properties. I also discuss alternative approaches used in the literature of factor inequality decompositions. I show that the RIF and the marginal and Shapley factor contributions are approximately equal in the case of the Mean Log Deviation, the index with the best additive decomposability properties, when the same normalization is used. In an empirical illustration, I use the approach to identify how the richest, highly educated, and urban population has disproportionally contributed to high and increasing inequality in Mozambique in recent years.

Keywords: inequality, decomposition, RIF, Shapley, marginal, Mozambique

JEL Classification: D31, D63, O15

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1 Introduction

The analysis of inequality by subpopulations has long been a key element for a better understanding of inequality levels and trends across countries. The aggregate decomposition of inequality between and within subpopulations obtained from various geographical and socioeconomic characteristics is a simple and powerful tool that helps us to identify the sources of inequality and its dynamics in different societies. The literature has already developed a sound analytical framework. Shorrocks (1984, 1988) has restricted the class of measures that are subgroup consistent, for which overall inequality increases with inequality in any group, ceteris paribus. This class is defined by the family of Generalized Entropy indices and any monotonic transformation, which includes the Atkinson family but not the popular Gini index. Scalar transformations of the Generalized Entropy family, alone, define a more specific class, the set of additively decomposable measures, in which inequality is the sum of inequality between groups and the weighted sum of inequality within groups (Shorrocks, 1980). Two members of this class proposed by Theil (1967)—the Mean Log Deviation and the Theil index—have become the most popular in empirical analysis of decompositions for their straight interpretation of the within-group component because the weights are respectively given by population and income shares. The Mean Log Deviation can be singled out as the only one whose decomposition does not depend on the path followed to break total inequality into its two components. Nevertheless, the decomposability properties of other indices have also been explored and used in empirical analyses. This is true of the role of overlapping among subpopulations in the Gini index (e.g. Bhattacharia and Mahalanobis, 1967 or Rao, 1969) and the multiplicative decomposability of the Atkinson family of (equality) indices (e.g. Blackorby et al, 1981). These aggregate decompositions, however, do not allow to explicitly identify the role of specific groups.

This paper contributes to this classical literature by proposing a simple framework into which we can go a step further and provide a more detailed decomposition by identifying the contribution of each subpopulation to overall inequality using the most popular indices.[1] In the case of additively decomposable indices, one can also identify the contribution to each component (between-group and within-group), mimicking the aggregate decomposition.

Following an individualistic approach, I define the contribution of a group to inequality as the sum of the contributions of its members, obtained by assessing the impact that a marginal increase in the proportion of people with a specific income would have on total inequality using the statistical notion of the Recentered Influence Function (RIF). This is not the only reasonable approach, though. For that reason, I also explore alternative approaches adapted from the factor inequality decomposition literature. I argue that contributions based on the natural decomposition rule of some indices are not especially attractive in this context. Then, I focus on the marginal contribution of a group, along an extension based on the Shapley decomposition that averages over all possible sequences in which groups are considered. For the especial case of the Mean Log Deviation—the index with the best additive decomposability properties—I show that the RIF, marginal and Shapley approaches are almost equivalent assuming a normalization property that is reasonable in the context of additively decomposable indices, i.e. a group with all incomes equal to the population mean has a null contribution to overall inequality. The three approaches may differ, though, for other indices of the same class.

[1] Although I focus here on inequality, the same approach can be used to explain the contribution of groups to other distributional statistics such as poverty, mobility, polarization, etc.

The contributions obtained with all three approaches will generally verify the usual properties of the target inequality indices (such as replication or scale invariance, or higher sensitivity to certain parts of the distribution). In this context, however, the linear RIF approach provides higher consistency in the decomposition because group contributions add up to total inequality and are independent of the level of disaggregation of groups. Furthermore, the RIF decomposition is straightforward to compute (including the standard errors).

The RIF contribution of a group can be obtained from the coefficients of group membership dummies in RIF regressions. Thus, the detailed RIF decomposition by subpopulations provides the researcher with a tool that is strongly complementary with one of the two main approaches used in the literature of regression-based inequality decompositions proposed in Firpo et al. (2007, 2009).[2] As Cowell and Fiorio (2011) have already noted, the two branches of the literature evolved highly disconnected one to each other.

I illustrate the potential of this approach with an analysis of the role of subpopulations defined by different household characteristics to explain the level and trend of consumption inequality in Mozambique, a low-income sub-Saharan African country that has shown a significant increase in inequality in recent years. The results will show the disproportional contributions to inequality and its increase over time of top percentiles, urban areas—especially the capital Maputo—and of households with heads having higher education.

The next section describes the proposed RIF decomposition, while the third section discusses other possible approaches. The fourth section introduces the data and an application to explain inequality in Mozambique in the 2008/09–2014/15 period. The last section concludes with final remarks.

2 The RIF decomposition of inequality by subpopulations

2.1 The total contribution of a subpopulation to inequality

Let us consider an exhaustive partition of a population into $K \geq 1$ disjoint groups, with superscript k referring to the kth group. Each group has a distribution $\boldsymbol{y^k} = (y_1^k, .., y_{n^k}^k)$, with population size n^k and mean income μ^k. The population of $n = \sum_{k=1}^{K} n^k$ individuals is represented by $\boldsymbol{y} = (\boldsymbol{y^1}, \dots, \boldsymbol{y^K})$ with mean income μ.

The Influence Function of an inequality index $IF(x; I(\boldsymbol{y}))$ measures the impact on inequality of marginally increasing the population mass at a certain income x (i.e. a small contamination) and has an expected value of zero. More formally, if $\boldsymbol{y_\varepsilon}$ is a mixture distribution assigning a probability $1 - \varepsilon$ to the original distribution $\boldsymbol{y}$ and ε to x (see Hampel, 1974):

$$IF(x; I(\boldsymbol{y})) = \frac{\partial}{\partial \varepsilon} I(\boldsymbol{y_\varepsilon})|_{\varepsilon=0} \text{ ; with } E(IF\big(x; I(\boldsymbol{y})\big) = 0$$

[2] These techniques allow to identify the contribution of several factors at the same time and/or to disentangle the compositional and structural effects in distributional changes. The other main approach is based on the factor decomposition of inequality using log-income regressions instead (Fields, 2003 and subsequent literature).

The Recentered Influence Function $RIF(x; I(\boldsymbol{y}))$ is obtained after recentering the IF (and its expected value) at the value of the target statistic (Firpo et al., 2007):

$$RIF(x; \mathrm{I}(\boldsymbol{y})) = I(\boldsymbol{y}) + IF(x; \mathrm{I}(\boldsymbol{y})); \text{ with } E(RIF(x; I(\boldsymbol{y})) = I(\boldsymbol{y}) .$$

The IF and RIF of several inequality measures have already been explored (Monti, 1991; Schechtman, 1991; Cowell and Flachaire, 2002 and 2007; or Essama-Nssah and Lambert, 2012). The corresponding expressions for discrete distributions for Gini and the Generalized Entropy family are shown in Table 1.

Let the contribution of the jth individual of group k to overall inequality $I(\boldsymbol{y})$, labelled as S_j^k, be the corresponding value of the per capita RIF estimated at y_j^k:[3]

$$S_j^k \equiv S_j^k(I(\boldsymbol{y})) = \frac{1}{n} RIF(y_j^k; I(\boldsymbol{y})).$$

The contribution of group k, labelled as S^k, is defined to be the sum of the contributions of its members:

$$S^k \equiv S^k(I(\boldsymbol{y})) = \sum_{j=1}^{n^k} S_j^k.$$

The relationship of S^k with the RIF regressions is straightforward, when group membership dummies ($\lambda^k = 1$ if an individual belongs to group k; 0 otherwise) are used as explanatory variables:

$$RIF(y_j^k; I(\boldsymbol{y})) = \sum_{k=1}^{K} \hat{\beta}^k \lambda^k; \text{ where } \hat{\beta}^k \equiv \bar{S}^k = \frac{1}{n^k} \sum_{j=1}^{n^k} RIF(y_j^k; I(\boldsymbol{y})).$$

The corresponding estimated coefficient of the RIF regression indicates the average RIF of the members of the group, $\bar{S}^k$, and the RIF group contribution can be obtained by multiplying this value by the group population share, after aggregating for all members:

$$S^k = \frac{1}{n} \sum_{j=1}^{n^k} \hat{\beta}^k \lambda_j^k = \frac{n^k}{n} \bar{S}^k.$$

In this context, in line with the RIF generalization of the Blinder (1973)-Oaxaca (1973) approach, a change in inequality could be decomposed into a compositional effect (i.e. changes in population sizes) and a structural effect (i.e. changes on the average RIF):[4]

$$\Delta I(\boldsymbol{y}) = \sum_{k=1}^{K} \Delta S^k = \sum_{k=1}^{K} \Delta(n_k/n)\bar{S}^k + \sum_{k=1}^{K} (n^k/n)\Delta\bar{S}^k.$$

Where Δ stands for the change in any statistic between the two distributions. This decomposition is obtained after just adding and subtracting a counterfactual distribution with the initial group sizes (characteristics) and the final average RIF (coefficients):

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[3] Cowell and Victoria-Feser (1996) have previously used the IF to study the robustness of various inequality indices to measurement error.

[4] The most popular case has been the decomposition of the difference in quantiles (known as unconditional quantile decomposition (Firpo et al. 2007, 2009), but the approach has also been applied to aggregate inequality measures like Gini (e.g. Gradín, 2016 and referenced literature) and others (e.g. Gradín, 2018).

$$I^c(\boldsymbol{y}) = \sum_{k=1}^{K} \frac{n^k}{n}(\bar{S}^k + \Delta\bar{S}^k).$$

It is interesting to note that the RIF approach applied to other simpler statistics provides very intuitive contributions. For example, the group size is the RIF contribution for the population size. The group mean and head-count ratio multiplied by the population share are the contributions for the corresponding population statistics (where z is a given poverty threshold):[5]

$$S^k(n) = n^k;$$

$$S^k(\mu) = \frac{n^k}{n}\mu^k;$$

$$S^k(H(\boldsymbol{y}; z)) = \frac{n^k}{n}H(\boldsymbol{y^k}; z).$$

The RIF contributions will inherit the properties of the corresponding inequality indices. For example, they will be invariant to replications of the entire population (population principle):

$$S^k(I(\boldsymbol{y})) = S^k\big(I(\boldsymbol{y}')\big) \text{ for any replication } \boldsymbol{y}' = (\boldsymbol{y}, \dots, \boldsymbol{y}).$$

They will also be invariant to the multiplication of all incomes in the population by the same factor (scale invariance):

$$S^k(I(\boldsymbol{y})) = S^k(I(\lambda\boldsymbol{y})) \text{ for any } \lambda > 0.$$

The RIF of inequality indices generally show a U-pattern with respect to income (e.g. Gradín, 2018). As a result, the contribution of a group to inequality will tend to be higher, ceteris paribus, the more people of this group have extreme incomes. The U-pattern will be asymmetric, though, reflecting the specific degree of sensitivity to income transfers that occur at different points of the distribution. The empirical section compares the cases of Gini, Theil and Mean Log Deviation indices for Mozambique.

Furthermore, the RIF decomposition verifies some important decomposability properties. Due to its linearity, the decomposition is consistent, overall inequality can be written as just the sum of all individual or group contributions:

$$I(\boldsymbol{y}) = \sum_{k=1}^{K} S^k = \sum_{k=1}^{K}\sum_{j=1}^{n^k} S_j^k.$$

Therefore, one might be interested in estimating the proportion of total inequality explained by each group, i.e. the relative total contribution:

$$s^k = S^k / I(\boldsymbol{y}).$$

Consistency also applies to further exhaustive partitions of group k, e.g. the contribution of a region in a country will be the sum of the contributions of the provinces integrating that region if they are computed separately.

[5] Given that $RIF(y_i; n) = 1$, $RIF(y_i; \mu) = y_i$, and $RIF(y_i; H(y_i, z)) = 1$ if $y_i < z$ ($= 0$ otherwise).

Additionally, S^k also verifies path independence with respect to the order in which the groups are considered and is invariant to the level of aggregation of other groups: the contribution of a group does not depend on how the other groups were clustered. For example, the contribution of a region does not change if the other regions are pooled together or considered as separate entities. Given the large variety of decomposability approaches, the next section investigates other decomposability features in the most common approach, additive decomposability.

The normalization and range of S^k are index-specific. For example, the Generalized Entropy family explored in more detail in the next section, verifies an intuitive normalization property, according to which $S^k = 0$ for a group with all incomes equal to the population mean ($y_j^k = \mu, \forall\, j = 1, \dots, n^k$), while $S^1 = I(\boldsymbol{y})$ if there is only one group in the population ($K = 1$). One special member of this family, the Mean Log Deviation—unlike other indices—also verifies an intuitive range property: S^k will always fall between 0 and $I(\boldsymbol{y})$.

2.2 The case of additively decomposable indices

An inequality index is additively decomposable (e.g. Shorrocks, 1984) if it can be written as the weighted sum of the inequality values calculated for the subpopulations (within-group inequality I_W) plus the contribution arising from differences between subpopulation means (between-group inequality I_B):

$$I(\boldsymbol{y}) = I_B + I_W.$$

Let $\mathbf{1_s}$ be a row vector of s ones and $\boldsymbol{\mu^k} = (\mu^1 \mathbf{1}_{\boldsymbol{n^1}}, \dots, \mu^K \mathbf{1}_{\boldsymbol{n^K}})$ the vector in which individual incomes y_j^k are replaced by their corresponding group means μ^k. Then, inequality within groups is defined as the level of inequality that goes away after equalizing incomes within groups, shifting from vector $\boldsymbol{y}$ to $\boldsymbol{\mu^k}$:

$$I_W = I(\boldsymbol{y}) - I(\boldsymbol{\mu^k}) = \textstyle\sum_{k=1}^{K} I(\boldsymbol{y^k}) w_I^k,$$

with w_I^k being index-specific weights that might be a function of group population shares n^k/n and relative incomes μ^k/μ.

Inequality between groups is the remaining level of inequality with $\boldsymbol{\mu^k}$:

$$I_B = I(\boldsymbol{\mu^k}).$$

The class of inequality indices that are additively decomposable and verify two basic properties, scale invariance and population replication, belongs (up to a scalar transformation) to the Generalized Entropy family $I_\alpha(\boldsymbol{y})$, including its limit when $\alpha = 0,1$ (Shorrocks, 1984). If subscript i identifies the ith individual of vector $\boldsymbol{y}$:

$$I_\alpha(\boldsymbol{y}) = \tfrac{1}{\alpha(\alpha-1)} \left[\tfrac{1}{n} \textstyle\sum_{i=1}^{n} \left(\tfrac{y_i}{\mu}\right)^\alpha - 1\right]; \text{ with } w_{I_\alpha}^k = \tfrac{n^k}{n} \left(\tfrac{\mu^k}{\mu}\right)^\alpha.$$

The most popular indices of this class are the two limit cases obtained for $\alpha = 0, 1$: The Mean Log Deviation (*M*) and Theil (*T*) indices, also previously characterized by Bourguignon (1979) and Foster (1983):

$$M \equiv I_0 = \frac{1}{n}\sum_{i=1}^{n} ln\frac{\mu}{y_i}; w_M^k = \frac{n^k}{n}.$$

$$T \equiv I_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{\mu} ln\frac{y_i}{\mu}; w_T^k = \frac{n^k}{n}\frac{\mu^k}{\mu}.$$

These are the two only cases in which the weights $w_{I_\alpha}^k$ used to construct the within-group component, respectively population and income shares, add up to 1. In other cases, the sum of weights depends on the between-group term, making the interpretation more difficult.

Among these two indices, *M* is known to be more sensitive to income transfers that occur in the lower tail of the distribution and has better decomposability properties. The decomposition of *M* is more genuine because the within-group component does not depend on group mean incomes. As a result, only the additive decomposition of *M* is independent of the path followed to define the two components (e.g. Chakravarty, 2009; Foster and Shneyerov, 2000).[6]

The contributions to total inequality are given by:

$$S^k(I_\alpha) = \frac{n^k}{n}\left[I_\alpha^k - \alpha\left(I_\alpha + \frac{1}{\alpha(\alpha-1)}\right)\frac{\mu^k-\mu}{\mu}\right]; \alpha \neq 0, 1.$$

$$S^k(M) = \frac{n^k}{n}\left[M^k + \frac{\mu^k-\mu}{\mu} + ln\frac{\mu}{\mu^k}\right].$$

$$S^k(T) = \frac{n^k}{n}\left[\left(\frac{\mu-\mu^k}{\mu}\right)(T+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu} + \frac{\mu^k}{\mu}T^k\right].$$

In parallel to the aggregate decomposition above, we can consistently define the contribution of each group to the within-group term as the contribution to inequality that goes away after equalizing incomes within groups. The contribution to inequality between groups is the remaining contribution, based on the aggregate impact on between-group inequality of marginally increasing the population with μ^k:

$$S_W^k = S^k\big(I(\boldsymbol{y})\big) - S^k\left(I\big(\boldsymbol{\mu^k}\big)\right).$$

$$S_B^k = S^k\left(I\big(\boldsymbol{\mu^k}\big)\right) = \frac{n_k}{n} RIF\left(\mu^k; I\big(\boldsymbol{\mu^k}\big)\right).$$

with $S^k = S_B^k + S_W^k$;

and $I_B = \sum_{k=1}^{K} S_B^k$ and $I_W = \sum_{k=1}^{K} S_W^k$.

[6] As an alternative to the case mentioned above, inequality between groups could be defined first, as the level of inequality that goes away when incomes are rescaled in each group so as to eliminate mean differences across groups. Within-group inequality would be then the remaining level when there is no inequality between groups. This would not affect the aggregate decomposition of *M*, neither the groups contribution to each component defined below.

In the two cases of interest, it is straightforward to obtain the corresponding expressions (summarized in Table 1). The between-group component:

$$S_B^k(M) = \frac{n^k}{n}\left(\frac{\mu^k-\mu}{\mu} + ln\frac{\mu}{\mu^k}\right);$$

$$S_B^k(T) = \frac{n^k}{n}\left[\left(\frac{\mu-\mu^k}{\mu}\right)(T_B + 1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu}\right].$$

The between-group component in both indices is the sum of two terms: the direct contribution to the indices, which is the function of the relative mean that goes into the summation in I_B (respectively $\frac{n^k}{n} ln\frac{\mu}{\mu^k}$ and $\frac{n^k}{n}\frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu}$), and the indirect contribution capturing the impact of each group in the population mean (and thus in the relative means of the other groups).

The between-group term S_B^k will be zero for *M* and *T* whenever $\mu^k = \mu$, and will monotonically increase the farther μ^k deviates from the critical value at which it reaches its minimum (μ in the case of *M*, $\mu e^{T_B} > \mu$ in the case of *T*). For a given relative mean income, the contribution of a group to this term will rise with the population size in both cases. Thus, the indices verify mean income and population share monotonicity properties. It is noticeable that $S_B^k(M)$, unlike $S_B^k(T)$, is independent of the overall between-group inequality level.

The corresponding within-group terms are given by:

$$S_W^k(M) = \frac{n^k}{n} M(\boldsymbol{y^k}).$$

$$S_W^k(T) = \frac{n^k}{n}\left[\frac{\mu^k}{\mu} T(\boldsymbol{y^k}) + T_W\left(\frac{\mu-\mu^k}{\mu}\right)\right].$$

The within-group term $S_W^k(M)$ is just the weighted level of group inequality. $S_W^k(T)$, however, is the weighted level of group inequality $w_T^k T^k$ plus another term. This second term captures the indirect effect that a higher population mass at μ^k has on the total within-group term, given that the weights depend on μ^k. Thus, $S_W^k(T)$, like $S_W^k(M)$, will monotonically increase with group size and inequality. But it will also be increasing with its relative mean if the group has higher inequality than the weighted average, $T(\boldsymbol{y^k}) > T_W$, or decreasing otherwise.[7] This is an unattractive property of the index decomposition related with its path dependence on the order in which T_W and T_B are defined. In particular, this implies that $T(\boldsymbol{y^k}) = 0$ is not enough condition for $S_W^k(T) = 0$, because group k still influences the within-group contributions of the other groups.

[7] In other words, the total effect of μ^k on T is given by $\frac{n^k}{n} RIF(\mu^k, T(\boldsymbol{y}))$, that can be split into the effect on the between-group component, $\frac{n^k}{n} RIF(\mu^k, T_B)$, and the effect on the within-group term, $\frac{n^k}{n} T_W\left(\frac{\mu-\mu^k}{\mu}\right)$.

In the same line as in *T*, the between-group contribution of the other members of the Generalized Entropy family (for $\alpha \neq 0, 1$) depends on overall between-group inequality, while the contribution to the within-group term depends on the group mean and overall within-group inequality:

$$S_B^k(I_\alpha) = \alpha\left(I_{B\,\alpha} + \frac{1}{\alpha(\alpha-1)}\right)\frac{n^k}{n}\frac{\mu-\mu^k}{\mu};$$

$$S_W^k(I_\alpha) = \frac{n^k}{n}\left[I_\alpha(\boldsymbol{y^k}) + \alpha\frac{n^k}{n}\left(1 - \left(\frac{\mu^k}{\mu}\right)^{1+\alpha}\right)I_{W\,\alpha}\right].$$

Table 1. Summary of RIF for Gini and Generalized Entropy Family (I_α)

	Gini	$\boldsymbol{I_\alpha}$	$\boldsymbol{M \equiv I_0}$	$\boldsymbol{T \equiv I_1}$
Index	$\frac{n+1}{n} - \frac{2}{n}\sum_{i=1}^{n}\left(\frac{n+1-i}{n}\right)\frac{y_i}{\mu}$	$\frac{1}{\alpha(\alpha-1)}\frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{y_i}{\mu}\right)^{\alpha}-1\right]$	$\frac{1}{n}\sum_{i=1}^{n} ln\frac{\mu}{y_i}$	$\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\mu} ln\frac{y_i}{\mu}$
$\boldsymbol{IF}(y_i, \boldsymbol{I})$	$2\frac{y_i}{\mu}\left(\frac{i}{n} - \frac{1+G}{2}\right) + 2\left(\frac{1-G}{2} - L_i\right)$	$\frac{1}{\alpha(\alpha-1)}\left[\left(\frac{y_i}{\mu}\right)^{\alpha}-1\right] - I_\alpha - \alpha\left(I_\alpha + \frac{1}{\alpha(\alpha-1)}\right)\frac{y_i-\mu}{\mu}$	$\frac{y_i-\mu}{\mu} + ln\frac{\mu}{y_i} - M$	$\frac{y_i}{\mu} ln\frac{y_i}{\mu} - \frac{y_i-\mu}{\mu} - \frac{y_i}{\mu}T$
$\boldsymbol{RIF}(y_i, \boldsymbol{I})$	$2\frac{y_i}{\mu}\left(\frac{i}{n} - \frac{1+G}{2}\right) + 2\left(\frac{1}{2} - L_i\right)$	$\frac{1}{\alpha(\alpha-1)}\left[\left(\frac{y_i}{\mu}\right)^{\alpha}-1\right] - \alpha\left(I_\alpha + \frac{1}{\alpha(\alpha-1)}\right)\frac{y_i-\mu}{\mu}$	$\frac{y_i-\mu}{\mu} + ln\frac{\mu}{y_i}$	$\frac{y_i}{\mu} ln\frac{y_i}{\mu} - (T+1)\left(\frac{y_i-\mu}{\mu}\right)$
$\boldsymbol{S^k}$	$\frac{2}{n}\sum_{j=1}^{n^k}\frac{j}{n}\frac{y_j}{\mu} - (1+G)\frac{n^k}{n}\frac{\mu^k}{\mu} + \frac{n^k}{n} - \frac{2}{n}\sum_{j=1}^{n^k} L_j^k$	$\frac{n^k}{n} I_\alpha^k - \alpha\left(I_\alpha + \frac{1}{\alpha(\alpha-1)}\right)\frac{n^k}{n}\frac{\mu^k-\mu}{\mu}$	$\frac{n^k}{n}\left[M^k + \frac{\mu^k-\mu}{\mu} + ln\frac{\mu}{\mu^k}\right]$	$\frac{n^k}{n}\left[\left(\frac{\mu-\mu^k}{\mu}\right)(T+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu} + \frac{\mu^k}{\mu}T^k\right]$
$\boldsymbol{S_B^k}$		$\alpha\left(I_{B_\alpha} + \frac{1}{\alpha(\alpha-1)}\right)\frac{n^k}{n}\frac{\mu-\mu^k}{\mu}$	$\frac{n^k}{n}\left(\frac{\mu^k-\mu}{\mu} + ln\frac{\mu}{\mu^k}\right)$	$\frac{n^k}{n}\left[\left(\frac{\mu-\mu^k}{\mu}\right)(T_B+1) + \frac{\mu^k}{\mu} ln\frac{\mu^k}{\mu}\right]$
$\boldsymbol{S_W^k}$		$\frac{n^k}{n} I_\alpha^k - \alpha\left(\frac{n^k}{n}\right)^2\left(\left(\frac{\mu^k}{\mu}\right)^{1+\alpha} - 1\right) I_{W_\alpha}$	$\frac{n^k}{n} M^k$	$\frac{n^k}{n}\left[\frac{\mu^k}{\mu}T^k + T_W\left(\frac{\mu-\mu^k}{\mu}\right)\right]$

Note: Incomes sorted from poorest to richest; $L_i = \frac{1}{n}\sum_{t=1}^{i}\frac{y_t}{\mu}$; $L_j^k = \frac{1}{n}\sum_{t=1}^{j}\frac{y_t^k}{\mu}$.

Source: Author's own construction.

3 A discussion of alternative approaches

In this section, I discuss alternative approaches to estimate the contribution of a subpopulation to inequality borrowed from the related literature that estimates the contribution of income factors (or sources). In general, the contributions will also inherit properties of the target inequality indices (e.g. replication and scale invariance).

The first approach, for the sake of simplicity will be referred to here as the marginal (factor) decomposition. It defines the contribution of a factor as the change in inequality after it has been removed. For example, comparing initial and final distributions has been a common way of assessing the redistributive effect of taxes (e.g. Kakwani, 1977). This is quite intuitive but has some limitations. It assumes a specific path for adding the different factors in which the target comes last. This is sometimes called the first-round effect. If other rounds or sequences are considered (the factor is the first, second, etc. to be added), the contributions will be path-dependent. Furthermore, it produces an inconsistent decomposition in which the contributions of all factors do not add up to total inequality (although they can be re-scaled accordingly). The same inconsistency applies to further disaggregation of that same factor.

A second approach is the Shapley decomposition (Chantreuil and Trannoy, 2013; Shorrocks, 2013). It is an extension of the marginal approach defined as the average of the marginal contributions obtained in all possible sequences in which each factor can be introduced. The Shapley decomposition is consistent and path-independent, but it will vary with the level of disaggregation of the various factors. This latter shortcoming might be overcome assuming some hierarchy in the involved factors,[8] although, in general, the Shapley decomposition (and its standard errors) might be cumbersome to compute in the presence of many groups and large samples.

In the analysis of income sources, there are at least two possible factors to be considered, each one imposing a different normalization property as it defines the situation in which the contribution is zero. The contribution of an income source (e.g. a tax or a benefit) to total inequality is the change in inequality after (i) removing the income source ('zero income' decomposition); or (ii) equalizing the income source among all individuals ('equalizing income' decomposition) (e.g. Sastre and Trannoy, 2002). In the same spirit, these cases are adapted here. I define the marginal contribution of a group as the change in inequality after (i) removing the group from the population ('zero subpopulation' decomposition); [9] or (ii) replacing the incomes of its members with the population mean ('equalizing subpopulation' decomposition). Note that the latter implies a double equalization, removing group inequality and removing the gap with the population mean, which seems reasonable in the context of additively decomposable indices.[10] Other alternatives seem less reasonable, such as (i) setting group incomes to zero; and (ii) replacing the incomes of group

[8] For example, using the Nested Shapley or the Owen decompositions, see the discussion in Chantreuil and Trannoy (2013) and in Charpentier and Mussard (2011).

[9] This has been implicitly used to measure the contribution of certain countries to global inequality (e.g. Sala-i-Martin, 2006).

[10] While in the case of the Atkinson family the mean should be replaced as a reference by the equally distributed equivalent income, in the case of Gini one needs to take also into account that a group with the population mean income and no inequality still has an impact on overall inequality through overlapping with the other groups. This implies that the normalization property with the marginal and Shapley approaches may need to be adapted to the specific index, while in the case of RIF it is directly inherited from the index.

members with the group mean. It would be hard to assume that a group does not contribute to inequality between groups in any of both situations.

Finally, we briefly consider a third alternative approach used in the factor decomposition literature, i.e. the natural decomposition rules of some inequality indices, such as the coefficient of variation (Shorrocks, 1982 showed it had implications for other measures under various assumptions), or Gini and Theil (Morduch and Sicular, 2002). In the first two cases, factor contributions can be further decomposed into the product of factor income shares, variability, and correlation with total incomes. This approach is index-specific (only applies to indices with a natural decomposition rule) and is somehow myopic, it does not fully account for the contribution of a factor as will be discussed below.

3.1 The zero-subpopulation decomposition

Let $\boldsymbol{y^{-k}} = \boldsymbol{y}\backslash\boldsymbol{y^k} = (\boldsymbol{y^1}, \dots, \boldsymbol{y^{k-1}}, \boldsymbol{y^{k+1}}, \dots, \boldsymbol{y^K})$ be the vector of the population remaining after removing group $\boldsymbol{k}$, with the superscript $-k$ applied to any statistic derived from it. The marginal zero subpopulation contribution of group $\boldsymbol{k}$ measures the impact of this removal on overall inequality:

$$\gamma^k = I(\boldsymbol{y}) - I(\boldsymbol{y^{-k}}).$$

This contribution can take positive or negative values. The normalization property behind this approach is that the contribution of a group is zero when it is not part of the population. For *M* and *T*, this is equivalent to the case in which the group has the same income mean and inequality of the rest, $\mu^k = \mu^{-k}$ and $I_\alpha(\boldsymbol{y^k}) = I_\alpha(\boldsymbol{y^{-k}})$ because:[11]

$$\gamma^k(I_\alpha) = I_\alpha\left((\mu^{-k}\mathbf{1}_{\boldsymbol{n^{-k}}}, \mu^k\mathbf{1}_{\boldsymbol{n^k}})\right) + w_{I_\alpha}^k\left(I_\alpha(\boldsymbol{y^k}) - I_\alpha(\boldsymbol{y^{-k}})\right) \text{ for } \alpha = 0{,}1.$$

Thus, if all groups had the same inequality and mean, all contributions to *M* or *T* would be zero and none of the observed inequality could be attributed to any group. Although this approach is interesting to assess the importance of a specific group on overall inequality, it does not seem adequate for attributing to each group its share of the total level of inequality, which is the objective here.

The Shapley contribution will be the average of the two possible marginal contributions, depending on whether the group or the rest of the population is the first to be removed.[12] The corresponding Shapley contributions is given by:

$$\gamma'^k = \frac{1}{2}\left(I(\boldsymbol{y}) + I(\boldsymbol{y^k}) - I(\boldsymbol{y^{-k}})\right).$$

---

[11] For any α in the Entropy family, the group inequality condition is that $w_{I_\alpha}^k I_\alpha(\boldsymbol{y^k}) = (1 - w_{I_\alpha}^{-k})I_\alpha(\boldsymbol{y^{-k}})$ because: $\gamma^k(I_\alpha) = I_\alpha\left((\mu^{-k}\mathbf{1}_{\boldsymbol{n^{-k}}}, \mu^k\mathbf{1}_{\boldsymbol{n^k}})\right) + w_{I_\alpha}^k I_\alpha(\boldsymbol{y^k}) - (1 - w_{I_\alpha}^{-k})I_\alpha(\boldsymbol{y^{-k}})$.

[12] For simplicity, I only consider here the case in which one group is compared against the rest of the population. Sastre and Trannoy (2002) provide the specific formula that can be used for the computation of a Shapley decomposition with *K* factors.

It is noteworthy that in both cases, in general, the differential between the contribution of one group and the rest of the population only depends on the differential in group inequality regardless of their population shares or relative incomes:

$$\gamma^k - \gamma^{-k} = \gamma'^k - \gamma'^{-k} = I(\boldsymbol{y^k}) - I(\boldsymbol{y^{-k}}).$$

Therefore, a highly unequal (equal) group will have a disproportionally larger positive (negative) contribution to inequality compared with other groups, even if its share of the population is almost zero.

3.2 The equalizing subpopulation decomposition

Alternatively, the contribution of a group can be defined as the impact of replacing their incomes with the population mean:

$$\delta^k = I(\boldsymbol{y}) - I\big((\boldsymbol{y^{-k}}, \mu \mathbf{1}_{\boldsymbol{n^k}})\big).$$

This implies the same normalization property verified by the RIF decomposition of the Generalized Entropy family of indices discussed above. Therefore, a null contribution to these indices is obtained if the group has the population mean and zero group inequality, the latter being a more demanding condition than just the same inequality as in the rest of the population, required in the previous approach. Note that, in this case, the differential in the contribution between two groups will depend on the inequality differential and on means and population shares:

$$\delta^k - \delta^{-k} = I\big((\mu \mathbf{1}_{\boldsymbol{n^{-k}}}, \boldsymbol{y^k})\big) - I\big((\boldsymbol{y^{-k}}, \mu \mathbf{1}_{\boldsymbol{n^k}})\big).$$

The contributions will not necessarily add up to the overall level of inequality, although they could be rescaled accordingly:

$$\tilde{\delta}^k = \delta^k \frac{I(\boldsymbol{y})}{\sum_{k=1}^{K} \delta^k}.$$

The corresponding Shapley contribution is given by:

$$\delta'^k = \frac{1}{2}\Big(I(\boldsymbol{y}) + I\big((\mu \mathbf{1}_{\boldsymbol{n^{-k}}}, \boldsymbol{y^k})\big) - I\big((\boldsymbol{y^{-k}}, \mu \mathbf{1}_{\boldsymbol{n^k}})\big)\Big).$$

In the case of additively decomposable indices, mimicking the aggregate decomposition (between group k and the rest of the population) we further split these contributions into the between-group and within-group components:

$$\delta^k(I_\alpha) = \delta_B^k(I_\alpha) + \delta_W^k(I_\alpha).$$

With:

$$\delta_B^k(I_\alpha) = I_\alpha\big((\mu^{-k} \mathbf{1}_{\boldsymbol{n^{-k}}}, \mu^k \mathbf{1}_{\boldsymbol{n^k}})\big) - I_\alpha\big((\mu^{-k} \mathbf{1}_{\boldsymbol{n^{-k}}}, \mu \mathbf{1}_{\boldsymbol{n^k}})\big).$$

$$\delta_W^k(I_\alpha) = w_{I_\alpha}^k I_\alpha\big(\boldsymbol{y^k}\big) + \frac{n^{-k}}{n} \pi(\alpha) I_\alpha\big(\boldsymbol{y^{-k}}\big).$$

where $\pi(\alpha) = \left(\left(\frac{\mu^{-k}}{\mu}\right)^\alpha - \left(\frac{\mu^{-k}}{\mu^*}\right)^\alpha\right)$ and $\mu^* = \frac{n^{-k}}{n}\mu^{-k} + \frac{n^k}{n}\mu$.

The marginal equalizing subpopulation contribution to *M* is approximately equal to the RIF contribution, with the same within-group term (given that $\pi(0) = 0$), and with a similar between-group component (provided $\frac{n^k}{n}\frac{\mu-\mu^k}{\mu}$ is small), using a well-known property of logarithms:[13]

$$\delta_B^k(M) = \frac{n^k}{n} ln \frac{\mu}{\mu^k} - ln\left(1 + \frac{n^k}{n}\frac{\mu-\mu^k}{\mu}\right) \approx S_B^k(M).$$

$$\delta_W^k(M) = \frac{n^k}{n} M^k = S_W^k(M).$$

A similar result can be obtained for the Shapley decomposition:

$$\delta'^k_B(M) = \frac{n^k}{n} ln \frac{\mu}{\mu^k} + \frac{1}{2} ln \left(\frac{1+\frac{n^k \mu^k - \mu}{n \quad \mu}}{1-\frac{n^k \mu^k - \mu}{n \quad \mu}}\right) \approx S_B^k(M).$$

$$\delta'^k_W(M) = \frac{n^k}{n} M(\boldsymbol{y^k}) = S_W^k(M);$$

While the direct contribution of each group to between-group inequality is the same in marginal, Shapley and RIF approaches, the indirect contribution (i.e. the impact of the group mean on the relative incomes of other groups) is non-linear in the case of the Shapley and marginal contributions, but linear in the case of RIF. Thus, the latter might be seen as a linear approximation of the former. It turns out that in the case of *M*, the index with the best additively decomposability properties, the RIF and the marginal and Shapley equalizing subpopulation decompositions are empirically equivalent, provided the $\frac{n^k}{n}\frac{\mu^k-\mu}{\mu}$ terms are small. One could only expect remarkable discrepancies in significant groups with an average income that is several times the population mean. The linearization that RIF imposes might be a reasonable assumption at least in the case of this index, bringing attractive properties at only a low price. These approaches may provide different results in other cases, like *T*, though. The empirical section will help to clarify this in practical terms.

3.3 Natural decomposition rules

If an index can be expressed as a weighted sum of incomes:

$$I(\boldsymbol{y}) = \sum_{i=1}^{n} a(\boldsymbol{y}) y_i,$$

then the contribution of a group can be defined by:

$$\tau^k(I) = \sum_{j=1}^{n^k} a(\boldsymbol{y}) y_j^k.$$

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[13] Note that $M\left((\mu^{-k}\mathbf{1}_{\boldsymbol{n^{-k}}}, \mu^k \mathbf{1}_{\boldsymbol{n^k}})\right) - M\left((\mu^{-k}\mathbf{1}_{\boldsymbol{n^{-k}}}, \mu \mathbf{1}_{\boldsymbol{n^k}})\right) = \left(\frac{n^k}{n} ln \frac{\mu}{\mu^k} + \frac{n^{-k}}{n} ln \frac{\mu}{\mu^{-k}}\right) - \left(\frac{n^k}{n} ln \frac{\mu^*}{\mu} + \frac{n^{-k}}{n} ln \frac{\mu^*}{\mu^{-k}}\right) = \frac{n^k}{n} ln \frac{\mu}{\mu^k} - ln \frac{\mu^*}{\mu} = \frac{n^k}{n} ln \frac{\mu}{\mu^k} - ln\left(1 + \frac{n^k}{n}\frac{\mu-\mu^k}{\mu}\right)$, and $ln(1+x) \approx x$ if x is small, with $x = \frac{n^k}{n}\frac{\mu-\mu^k}{\mu}$. The two approaches will be closer if the marginal contributions are rescaled to add up to total inequality, as shown in the empirical illustration.

M does not belong to this class of indices. In the case of *T*, $a(\boldsymbol{y}) = \frac{1}{n\mu}\ln\frac{y_j^k}{\mu}$ and the contribution is given by:

$$\tau^k(T) = \frac{1}{n}\sum_{j=1}^{n^k}\frac{y_j^k}{\mu}\ln\frac{y_j^k}{\mu} = \frac{n^k}{n}\frac{\mu^k}{\mu}T^k + \frac{n^k}{n}\frac{\mu^k}{\mu}\ln\frac{\mu^k}{\mu} = S^k(T) - \frac{n^k}{n}\left(\frac{\mu-\mu^k}{\mu}\right)(T+1).$$

That is, the contribution does not measure the total effect of group incomes. It only considers the direct effect on total inequality, but not the indirect effect on the function $a(\boldsymbol{y})$. This is true of the natural decomposition of any index. In the case of *T*, this means to ignore the impact of the average group mean on the relative incomes of the rest of the population – that affects both terms, inequality between groups and within groups, the latter through changes in group income shares. Given that the RIF approach is more general and accounts for both direct and indirect effects, this approach does not seem especially attractive in this context.

4 Empirical analysis

4.1 Data

To illustrate the potential of the approach, I use the two most recent household budget surveys collected by the *Instituto Nacional de Estatística* (INE) in Mozambique: the *Inquéritos ao Orçamento Familiar* (IOF 2008/09 and 2014/15). Wellbeing is approximated using daily real per capita consumption (MEF/DEEF, 2016), i.e. nominal consumption adjusted to correct for seasonal variation in prices, deflated by dividing by the contemporary poverty line.

The sample of the first survey consists of 10,832 households (51,177 individuals) interviewed between 2008 and 2009. The most recent survey is a pool of about 11,000 households interviewed up to three times between 2014 and 2015: 11,505 households (58,342 individuals) in the first quarter, 10,372 (50,770) in the second, and 11,315 (55,198) in the third one.

For the analysis, I considered four types of subpopulations: consumption percentile groups, area of residence (rural or urban), province, and level of education attained by the household head. The analysis is focused on the *M* index and the RIF decomposition, because of its attractive decomposability properties, although I also provide results for other indices and approaches.

4.2 Total inequality and the contribution by percentile

Inequality increased in Mozambique between 1996/97 and 2002/03, followed by stagnation between 2002/03 and 2008/09, to raise again between 2008/09 and 2014/15 (Gradín and Tarp, 2017). The increase during the last period was of 0.053 with Gini (13 per cent) and 0.078 with *M* (from 0.303 to 0.381, 26 per cent), as reported in Table 2 also for other indices (the change was robust due to Lorenz dominance).

Figure 1 shows how the RIF values of different indices vary according to relative consumption. The RIF of *M* is independent of the distribution, while for *T* and Gini, they are based on the most recent distribution in Mozambique. The U-pattern becomes clear, and the higher sensitivity of *M* and, especially, *T* to higher relative consumption values compared with Gini (known to be more sensitive to central values), as well as the higher sensitivity of *M* to values close to 0. Figure 2 shows, accordingly, the contribution by percentile group to inequality in Mozambique in 2014/15

as measured by these three indices for comparison.[14] Table 3 summarizes the results for specific percentile ranges for several indices.

The largest contributions to *M* came from the highest and, to a lesser extent, from the lowest percentile groups. This has the implication that in the analysis by other subpopulation groups, in the next epigraph, the average contribution will tend to be larger for those groups that are overrepresented at the top of the consumption scale. However, it is also noticeable that the range of lower percentiles with a significant contribution is longer and thus large groups with most of the population at the bottom of consumption levels will also have an important contribution to overall *M* inequality. On the other hand, people (and groups) around the mean consumption level (around percentile 73) do not contribute at all to inequality with *M*.

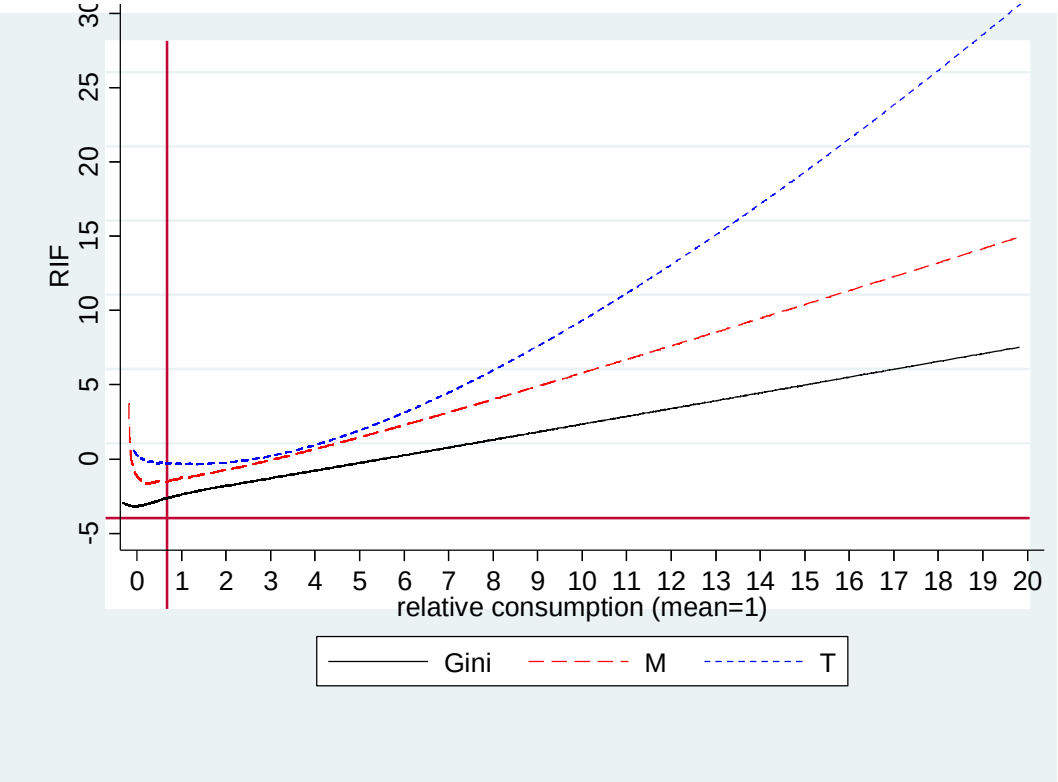
More specifically, the contribution of the top 5 per cent population to *M* is almost 42 per cent, not only much higher than their demographic weight, but also higher than their contribution to total consumption (28.5 per cent). The contribution of the bottom 5 per cent is also above its demographic weight, 14.5 per cent (with only 0.8 per cent of total consumption). That is, both extremes, with only 10 per cent of the population altogether, contribute to almost 57 per cent of total inequality. On the opposite side, the contributions fall below 1 per cent between percentiles 20th and 93rd.

As for the other indices, the relative contributions of both poorest and richest percentiles are smaller with the Gini index (e.g. 8.4 and 24 per cent for the bottom and top 5 per cent, respectively), with the contribution of the rich below their contribution to total consumption. While the contribution of the poorest percentiles is also smaller with *T* (near 10 per cent) than with *M*, the contribution of the very richest is even higher: 45.6 per cent, but not as much as for I_2. The Atkinson family presents the expected results, lower contribution of the richest and higher of the poorest for higher values of the inequality aversion parameter.

When it comes to explaining the trend in inequality over time between 2008/09–2014/15 (Figure 3 and Table 4), about 60 per cent of the increase in *M* was associated with a larger contribution of the top 5 per cent (42 per cent in the top 1 per cent), the result of a clear pro-rich growth pattern in consumption. On the other side, the bottom 5 per cent of the consumption distribution only contributed with less than 1 per cent of the inequality increase. Contributions to the change in inequality were indeed slightly negative in the first two percentiles, as they were between percentiles 74 and 91. The contribution of the rich is much smaller but still significant in the case of Gini (38 per cent) and declines with inequality aversion in the case of the Atkinson family.

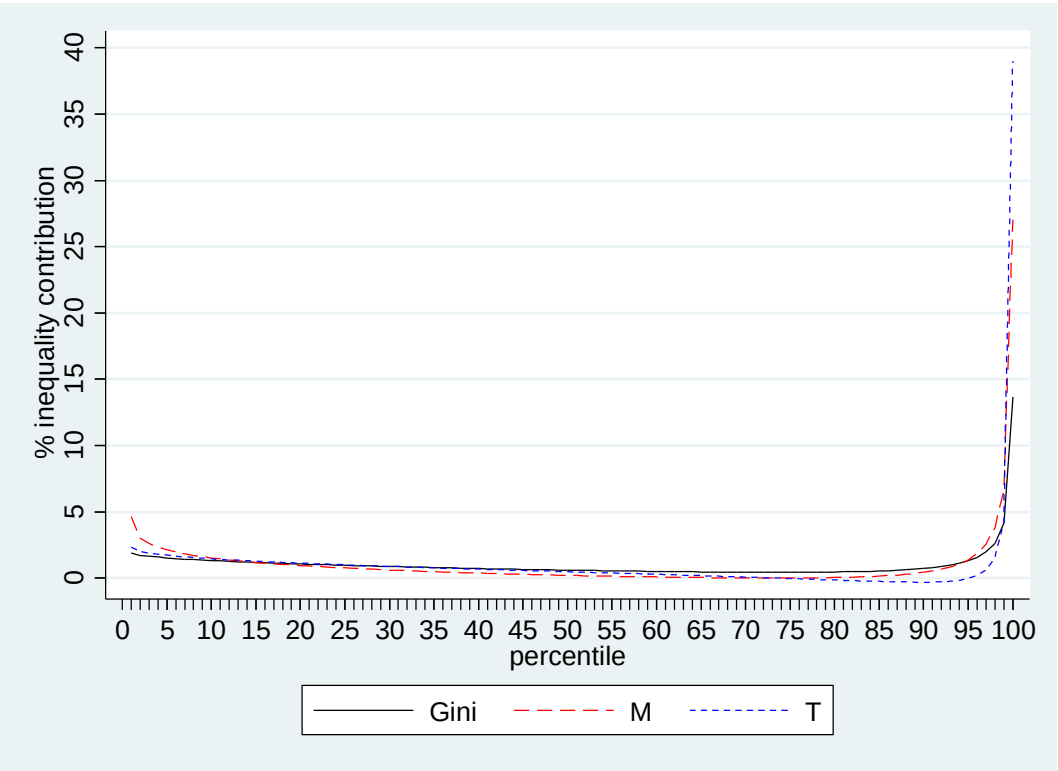
[14] Previous analyses of RIF functions by percentiles can be found in Gradín (2016 and 2018) or Davies et al. (2017).

Figure 1: RIF of relative consumption for *M*, *T* and Gini



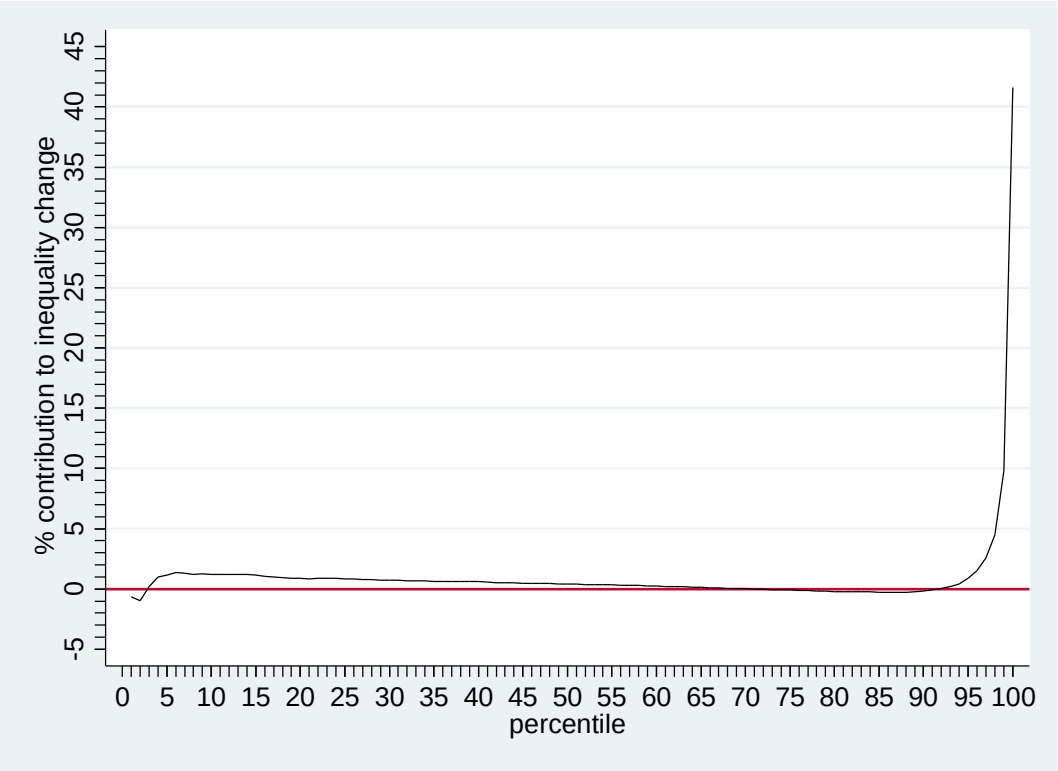
Note: The RIF of *M* is independent of the distribution. RIF of *T* (*T*=0.520) and Gini are computed for Mozambique 2014/15. Source: Author's calculations based on IOF 2014/15.

Figure 2: Relative contributions to inequality by percentile group, 2014/15



Source: Author's calculations based on IOF 2014/15.

Figure 3: Relative contributions to inequality increase (*M*) by percentile group, 2008/09–2014/15



Source: Author's calculations based on IOF 2008/09–2014/15.

Table 2: Consumption inequality

	Per capita Consumption (Meticais)	Gini	I_{-1}	I_0=M	I_1=T	I_2	$A_{.25}$	$A_{.50}$	$A_{.75}$	A_1	A_2
2008/09	23.8	0.415	0.409	0.303	0.367	0.887	0.082	0.150	0.209	0.262	0.450
2014/15	47.1	0.468	0.532	0.381	0.520	2.242	0.111	0.194	0.261	0.317	0.516

Source: Author's calculations based on IOF 2008/09–2014/15.

Table 3: Relative RIF contributions to inequality by percentile range in 2014/15

Percentile range	**Population share**	**Consumption share**	**Inequality shares**									
			Gini	**I_{-1}**	**I_0=M**	**I_1=T**	**I_2**	**$A_{.25}$**	**$A_{.50}$**	**$A_{.75}$**	**A_1**	**A_2**
Bottom 5	5	0.8	8.4	33.6	14.7	9.7	9.3	10.4	11.2	12.0	13.0	18.9
6-25	20	6.5	23.7	15.0	24.5	25.6	29.1	25.4	25.0	24.5	23.7	17.6
26-75	50	34.3	31.1	-4.5	12.5	23.2	32.9	21.3	20.0	19.3	19.2	23.6
76-95	20	30.0	12.9	13.9	6.5	-4.1	-18.3	-0.8	2.7	5.9	8.9	17.0
Top 5	5	28.5	24.0	42.0	41.9	45.6	47.0	43.7	41.1	38.2	35.3	22.9
Total	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***

Source: Author's calculations based on IOF 2014/15.

Table 4: Relative RIF contributions to inequality increase between 2008/09 and 2014/15, by percentile range

Percentile group	Population share	%ΔConsumption	Inequality share									
			Gini	$I_{.1}$	I_0=M	I_1=T	I_2	$A_{.25}$	$A_{.50}$	$A_{.75}$	A_1	A_2
Bottom 5	5	0.7	-0.1	31.1	0.8	4.5	8.5	3.4	1.8	-0.3	-3.1	3.2
6-25	20	5.6	15.1	11.3	21.5	21.6	28.6	20.8	20.3	19.9	19.8	15.6
26-75	50	30.6	39.9	-11.0	19.5	32.0	36.9	30.9	30.7	31.3	32.6	40.7
76-95	20	29.1	7.1	5.9	-1.8	-8.7	-18.9	-6.2	-3.2	-0.3	2.5	10.7
Top 5	5	34.0	38.1	62.7	60.1	50.7	45.0	51.2	50.5	49.4	48.2	29.8
Total	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***

Source: Author's calculations based on IOF 2008/09–2014/15.

4.3 Analysis by other subpopulations

Table 5 reports the conventional aggregate additive decomposition of *M* in 2014/15 by province, area of residence and head's education (rows labelled as 'All'). For example, total inequality (0.381) can be split into the two corresponding between-province (0.067, 17.5 per cent) and within-province (0.314, 82.5 per cent) terms. Table 5 also reports the more detailed RIF decomposition, showing that the capital Maputo city, representing only 5 per cent of the total population or 14 per cent of total consumption, disproportionally contributed with 17 per cent of total inequality in that year, of which almost 10 percentage points were through the between-group component (56 per cent of the overall between-group component). The large contribution of Maputo city was the result of combining two facts. It is the richest among all provinces (per capita consumption is 2.8 times the national average), influencing its between-group contribution. It has the highest level of inequality too (within-group term). Something similar is found in Maputo province, even if to a lesser extent. With near 7 per cent of the population (and 11 per cent of consumption) Maputo province contributes with 9 per cent of total inequality, 3 per cent through the between-group component, and 6 per cent through the within-group component.

On the other hand, northern provinces of Zambézia and Nampula show similar contributions of 16 and 17 per cent of total inequality, respectively. But in these cases, the large contribution is the result of these provinces representing near 20 per cent of the population each. Indeed, they show an average RIF below the mean because their relative consumption levels are the lowest (78 and 76 per cent of the average), with intermediate inequality levels (0.304 and 0.291). Thus, their contributions came almost entirely from the within-group component, to which they are the largest contributors (near 15 per cent each) given their large population sizes.

In the middle between both extremes, other provinces like Niassa, Sofala, Inhambane, or Gaza, have a total contribution only slightly below their corresponding population shares. Not surprisingly, some provinces, with consumption levels around 87-103 per cent of the national average have a negligible contribution to between-group inequality, while only the richest ones, as mentioned above Maputo city and province, and the poorest (Niassa, Nampula and Zambézia with 1.3-17 per cent) report substantial contributions through this component. Similarly, and consistently with the discussion above, urban and rural areas contribute about a half each to total inequality, although in the case of urban areas this is due to a higher average RIF, because they make up less than a third of the population.

The analysis by the level of education attained by the household head shows that the small group of people in households whose head had some college contributed with 18 per cent of inequality, despite being only 2.5 per cent of the population. This contribution occurs mostly through the between-group component (14 per cent of total inequality), because mean consumption is almost

5 times the average. This group is also the most unequal, but given its small size only makes a small contribution to the within-group component (4 per cent, still above its population share). Educational groups below secondary education, on the contrary, contribute to inequality below their share of the population.

The role of each of these subpopulations in explaining the increase of 0.078 in overall inequality between 2008/09 and 2014/15 is reported in Table 6.[15] Overall, more than half the increase in inequality was between provinces, but also with a substantial increase within provinces. Maputo city and province, alone, are responsible for two thirds of the increase in inequality over this period (39.8 and 25.3 respectively). This is primarily the result of their higher contribution to inequality between groups (due to the significant increase in per capita consumption and thus in their average contribution to this term, especially in Maputo city). To a lesser extent, it is also the result of their higher contribution to the within-group term (due to the substantial increase in inequality within these two areas). Zambézia also explains 15 per cent of the increase in inequality, in this case almost entirely because of the increase in the level of inequality in this province. Niassa and Sofala show negative contributions due to the fall in inequality, while the negative contribution of Gaza is associated with its loss of population.

In the same line, only about 20 per cent of the increase in inequality is between urban and rural areas. Urban areas explained 84 per cent of the total increase in overall inequality, but in this case, it is inequality within these areas that dominates (63 of the increase, compared with 21 per cent in the between-group case). Furthermore, more than half the increase in inequality was between educational groups, with the responsibility being more bipolarized in this case. The lowest educated group contributes to 43 per cent of the increase, especially due to the within-group component (30 per cent), the highest educated to another 44 per cent, especially to the between-group term (34 per cent).

Table 7 shows that, as expected, the contributions to *M* computed using the marginal and Shapley equalizing subpopulation approaches are almost identical to RIF contributions described above, with only non-negligible differences for the contribution of the very richest (top 5 percentile group).

The results for RIF using *T*, instead of *M*, to measure inequality are reported in Table 8. In this case, not surprisingly, the contribution of richest groups to the within-group inequality tends to be higher than with *M* because of their higher relative weight (income share instead of population share). For example, the total contribution of Maputo city is now 21 per cent, 9 per cent through the between-group component, 12 through the within-group. In the same line, the contribution to the intra-group component made by households whose head has some college education has now doubled (7 per cent). The marginal and, to a lesser extent, the Shapley contributions to *T* are also not too far from the RIF values (Table 7), although in this case they diverge more than they did for *M*. Especially, they tend to increase even further the contribution of richest groups, like the top 5 percentile groups or urban areas (and decrease those of poorest groups).

[15] In Gradín and Tarp (2017) using RIF regressions controlling for several characteristics at the same time, higher education of household heads was identified as the most relevant driving factor of increasing inequality, along more heads working in the private non-subsistence sector of the economy.

Table 5: RIF decomposition of inequality (*M*) in Mozambique by subpopulations in 2014/15

	Descriptive (%)		Inequality											
	Population	Consumpt.	Total	Average contribution			Total contribution							
				Total	Between -Group	Within -Group	Overall		Between-Group			Within-Group		
Province	n_k/n	μ_k/μ	$M(y^k)$	$\bar{S}^k$	$\bar{S}_B^k$	$\bar{S}_W^k$	S^k	$\%M$	S_B^k	$\%M$	$\%M_B$	S_W^k	$\%M$	$\%M_W$
Niassa	6.4	66.1	0.267	0.342	0.075	0.267	0.022	5.7	0.005	1.3	7.2	0.017	4.5	5.4
Cabo Delgado	7.4	87.8	0.243	0.251	0.008	0.243	0.018	4.8	0.001	0.2	0.9	0.018	4.7	5.7
Nampula	19.5	77.7	0.304	0.333	0.029	0.304	0.065	17.0	0.006	1.5	8.6	0.059	15.5	18.8
Zambezia	18.8	76.0	0.291	0.325	0.035	0.291	0.061	16.0	0.006	1.7	9.7	0.055	14.3	17.4
Tete	9.8	97.6	0.247	0.247	0.000	0.247	0.024	6.3	0.000	0.0	0.0	0.024	6.3	7.7
Manica	7.5	93.2	0.259	0.261	0.002	0.259	0.020	5.1	0.000	0.0	0.3	0.019	5.1	6.2
Sofala	7.9	102.7	0.382	0.383	0.000	0.382	0.030	7.9	0.000	0.0	0.0	0.030	7.9	9.6
Inhambane	5.8	95.0	0.340	0.341	0.001	0.340	0.020	5.2	0.000	0.0	0.1	0.020	5.2	6.3
Gaza	5.5	89.8	0.345	0.350	0.006	0.345	0.019	5.1	0.000	0.1	0.5	0.019	5.0	6.1
Maputo province	6.6	169.4	0.376	0.542	0.167	0.376	0.036	9.3	0.011	2.9	16.4	0.025	6.5	7.8
Maputo City	4.9	280.1	0.583	1.354	0.771	0.583	0.066	17.3	0.038	9.8	56.3	0.028	7.5	9.0
All	***100***	***100***	***0.381***	***0.381***	***0.067***	***0.314***	***0.381***	***100***	***0.067***	***17.5***	***100***	***0.314***	***82.5***	***100***
Area														
Rural	68.3	78.8	0.243	0.270	0.026	0.243	0.184	48.3	0.018	4.7	41.2	0.166	43.6	49.3
Urban	31.7	145.7	0.541	0.622	0.081	0.541	0.197	51.7	0.026	6.7	58.8	0.171	44.9	50.7
All	***100***	***100***	***0.381***	***0.381***	***0.043***	***0.338***	***0.381***	***100***	***0.043***	***11.4***	***100***	***0.338***	***88.6***	***100***
Education														
Less than primary	30.5	72.4	0.285	0.333	0.047	0.285	0.101	26.6	0.014	3.8	15.2	0.087	22.8	30.3
Lower Primary	43.9	82.1	0.247	0.265	0.018	0.247	0.116	30.5	0.008	2.1	8.4	0.108	28.4	37.8
Upper Primary	13.9	105.9	0.300	0.302	0.002	0.300	0.042	11.0	0.000	0.1	0.2	0.042	11.0	14.6
Lower Secondary	4.1	139.8	0.338	0.401	0.063	0.338	0.016	4.3	0.003	0.7	2.7	0.014	3.6	4.8
Upper Secondary	3.3	207.1	0.432	0.775	0.343	0.432	0.026	6.8	0.011	3.0	12.2	0.014	3.8	5.0
Technical	0.7	250.9	0.470	1.059	0.589	0.470	0.008	2.0	0.004	1.1	4.5	0.003	0.9	1.2
Some college/ normal	2.5	469.1	0.574	2.719	2.146	0.574	0.068	17.8	0.053	14.0	56.7	0.014	3.8	5.0
Unknown	1.1	94.7	0.334	0.335	0.001	0.334	0.004	0.9	0.000	0.0	0.0	0.004	0.9	1.2
All	***100***	***100***	***0.381***	***0.381***	***0.094***	***0.287***	***0.381***	***100***	***0.094***	***24.8***	***100***	***0.287***	***75.2***	***100***

Source: Author's calculations based on IOF 2014/15.

Table 6: RIF decomposition of the change in inequality (*M*) in Mozambique by subpopulations between 2008/09 and 2014/15

	Descriptive (%)		Inequality									
	Population	Consumption		Average contribution			Total contribution					
				Total	Between -Group	Within -Group	Total		Between-Group		Within-Group	
Province	$\Delta(n_k/n)$	$\Delta(\mu_k/\mu)$	$\Delta M(y^k)$	$\Delta\bar{S}^k$	$\Delta\bar{S}_B^k$	$\Delta\bar{S}_W^k$	ΔS^k	%ΔM	ΔS_B^k	%ΔM	ΔS_W^k	%ΔM
Niassa	0.5	-68.9	-0.078	-0.053	0.025	-0.078	-0.002	-2.0	0.002	2.3	-0.003	-4.3
Cabo Delgado	-0.5	-20.6	0.046	0.051	0.005	0.046	0.003	3.6	0.000	0.4	0.002	3.2
Nampula	0.3	-22.9	0.001	0.030	0.029	0.001	0.007	8.8	0.006	7.4	0.001	1.5
Zambezia	-0.2	-2.3	0.060	0.066	0.007	0.060	0.012	15.2	0.001	1.6	0.011	13.6
Tete	0.8	0.3	0.039	0.039	0.000	0.039	0.005	7.0	0.000	0.0	0.005	7.0
Manica	0.5	7.9	0.049	0.040	-0.010	0.049	0.004	5.3	-0.001	-0.8	0.005	6.1
Sofala	-0.2	8.3	-0.038	-0.040	-0.001	-0.038	-0.004	-5.2	0.000	-0.1	-0.004	-5.1
Inhambane	-0.3	-3.5	0.082	0.083	0.001	0.082	0.004	5.2	0.000	0.1	0.004	5.1
Gaza	-0.8	5.7	0.013	0.005	-0.009	0.013	-0.002	-3.0	-0.001	-0.7	-0.002	-2.3
Maputo Province	0.3	74.6	0.125	0.290	0.166	0.125	0.020	25.3	0.011	14.0	0.009	11.3
Maputo City	-0.4	95.2	0.148	0.685	0.537	0.148	0.031	39.8	0.025	32.5	0.006	7.3
All	***0.0***	***0.0***	***0.078***	***0.078***	***0.044***	***0.034***	***0.078***	***100***	***0.044***	***56.5***	***0.034***	***43.5***
Area												
Rural	-1.3	-9.6	0.003	0.022	0.019	0.003	0.012	15.7	0.013	16.5	-0.001	-0.9
Urban	1.3	19.3	0.139	0.190	0.051	0.139	0.066	84.3	0.016	21.2	0.049	63.1
All	***0.0***	***0.0***	***0.078***	***0.078***	***0.029***	***0.049***	***0.078***	***100***	***0.029***	***37.7***	***0.049***	***62.3***
Education												
Less than primary	5.4	-10.2	0.032	0.062	0.030	0.032	0.034	43.1	0.010	12.9	0.023	30.1
Lower Primary	-11.4	-5.7	0.018	0.028	0.010	0.018	-0.015	-18.9	0.004	4.5	-0.018	-23.4
Upper Primary	1.3	-6.9	0.026	0.021	-0.006	0.026	0.007	8.5	-0.001	-0.9	0.007	9.4
Lower Secondary	1.1	-21.6	0.015	-0.057	-0.072	0.015	0.002	3.2	-0.002	-2.0	0.004	5.1
Upper Secondary	1.8	-24.5	0.057	-0.076	-0.133	0.057	0.013	16.2	0.004	5.2	0.009	11.0
Technical	-0.1	12.3	0.138	0.211	0.073	0.138	0.001	0.8	0.000	-0.1	0.001	0.8
Some college/ normal	1.3	-8.5	0.023	-0.044	-0.067	0.023	0.034	44.2	0.027	34.4	0.008	9.8
Unknown	0.6	32.0	0.152	0.059	-0.092	0.152	0.002	3.1	0.000	-0.5	0.003	3.6
All	***0.0***	***0.0***	***0.078***	***0.078***	***0.042***	***0.036***	***0.078***	***100***	***0.042***	***53.5***	***0.036***	***46.5***

Source: Author's calculations based on IOF 2008/09–2014/15.

Table 7: Relative Decompositions (per cent) of overall inequality (*M* and *T*) by subpopulation, Mozambique 2014/15

	M			*T*		
Percentiles	**RIF** S^k	**Marginal*** $\widetilde{\delta}^k$	**Shapley** δ'^k	**RIF** S^k	**Marginal*** $\widetilde{\delta}^k$	**Shapley** δ'^k
Bottom 5%	14.7	12.9	14.5	9.7	8.0	7.3
6-25	24.5	23.1	23.8	25.6	20.2	17.5
26-75	12.5	13.3	11.7	23.2	18.5	13.5
76-95	6.5	6.8	7.0	-4.1	-2.9	1.8
Top 5%	41.9	43.7	43.0	45.6	56.3	59.9
All	***100***	***100***	***100***	***100***	***100***	***100***
Province						
Niassa	5.7	5.7	5.7	5.3	5.1	4.1
Cabo Delgado	4.8	4.8	4.8	4.7	4.5	4.2
Nampula	17.0	17.0	17.0	20.0	18.9	17.8
Zambezia	16.0	16.0	16.0	14.8	14.0	12.5
Tete	6.3	6.2	6.3	5.2	5.0	5.1
Manica	5.1	5.0	5.1	4.5	4.4	4.3
Sofala	7.9	7.8	7.9	7.3	7.2	7.4
Inhambane	5.2	5.1	5.2	4.7	4.6	4.5
Gaza	5.1	5.0	5.1	4.6	4.5	4.3
Maputo province	9.3	9.4	9.4	7.8	8.2	10.1
Maputo City	17.3	18.0	17.4	21.2	23.5	25.6
All	***100***	***100***	***100***	***100***	***100***	***100***
Area						
Rural	48.3	48.2	48.1	46.6	39.5	38.7
Urban	51.7	51.8	51.9	53.4	60.5	61.3
All	***100***	***100***	***100***	***100***	***100***	***100***
Education						
Less than primary	26.6	26.7	26.5	27.5	25.8	23.1
Lower Primary	30.5	30.4	30.5	28.7	26.9	24.6
Upper Primary	11.0	10.7	11.0	9.1	9.1	9.6
Lower Secondary	4.3	4.2	4.3	3.2	3.2	4.0
Upper Secondary	6.8	6.8	6.8	6.8	7.2	8.7
Technical	2.0	2.0	2.0	1.9	1.9	2.4
Some college/ normal	17.8	18.4	17.9	22.1	25.1	26.8
Unknown	0.9	0.9	0.9	0.8	0.8	0.8
All	***100***	***100***	***100***	***100***	***100***	***100***

Note: (*) Marginal approach normalized so that group contributions add up to total inequality.

Source: Author's calculations based on IOF 2014/15.

Table 8: RIF decomposition of inequality (*T*) in Mozambique by subpopulations in 2014/15

	Inequality											
	Index	Average contribution			Total contribution							
		Total	Between -Group	Within -Group	Overall		Between-Group			Within-Group		
					Abs.	%	Abs.	%		Abs.	%	
Province	$T(y^k)$	$\bar{S}^k$	$\bar{S}_B^k$	$\bar{S}_W^k$	S^k	$\%T$	S_B^k	$\%T$	$\%T_B$	S_W^k	$\%T$	$\%T_W$
Niassa	0.280	0.427	0.093	0.334	0.027	5.3	0.006	1.1	7.2	0.021	4.1	4.9
Cabo Delgado	0.295	0.331	0.018	0.313	0.024	4.7	0.001	0.3	1.6	0.023	4.4	5.3
Nampula	0.502	0.533	0.045	0.488	0.104	20.0	0.009	1.7	10.7	0.095	18.3	21.7
Zambezia	0.333	0.410	0.051	0.358	0.077	14.8	0.010	1.9	11.7	0.067	12.9	15.4
Tete	0.269	0.275	0.002	0.273	0.027	5.2	0.000	0.0	0.3	0.027	5.1	6.1
Manica	0.298	0.316	0.008	0.308	0.024	4.5	0.001	0.1	0.7	0.023	4.4	5.3
Sofala	0.481	0.481	-0.002	0.483	0.038	7.3	0.000	0.0	-0.2	0.038	7.3	8.7
Inhambane	0.410	0.417	0.005	0.412	0.024	4.7	0.000	0.1	0.4	0.024	4.6	5.5
Gaza	0.415	0.431	0.014	0.418	0.024	4.6	0.001	0.1	0.9	0.023	4.4	5.3
Maputo province	0.461	0.618	0.142	0.477	0.041	7.8	0.009	1.8	11.3	0.031	6.0	7.1
Maputo City	0.755	2.262	0.935	1.327	0.110	21.2	0.046	8.8	55.3	0.065	12.4	14.8
All	***0.520***	***0.520***	***0.082***	***0.438***	***0.520***	***100***	***0.082***	***15.8***	***100***	***0.438***	***84.2***	***100***
Area												
Rural	0.279	0.354	0.034	0.321	0.242	46.6	0.023	4.5	50.8	0.219	42.1	46.2
Urban	0.703	0.878	0.071	0.807	0.278	53.4	0.022	4.3	49.2	0.255	49.1	53.8
All	***0.520***	***0.520***	***0.046***	***0.474***	***0.520***	***100***	***0.046***	***8.8***	***100***	***0.474***	***91.2***	***100***
Education												
Less than primary	0.390	0.468	0.079	0.389	0.143	27.5	0.024	4.6	18.2	0.119	22.8	30.6
Lower Primary	0.280	0.340	0.041	0.299	0.149	28.7	0.018	3.4	13.5	0.131	25.2	33.9
Upper Primary	0.349	0.340	-0.006	0.346	0.047	9.1	-0.001	-0.2	-0.6	0.048	9.3	12.5
Lower Secondary	0.387	0.404	0.018	0.386	0.017	3.2	0.001	0.1	0.5	0.016	3.0	4.1
Upper Secondary	0.571	1.063	0.294	0.768	0.036	6.8	0.010	1.9	7.4	0.026	4.9	6.6
Technical	0.532	1.349	0.599	0.751	0.010	1.9	0.004	0.8	3.2	0.005	1.0	1.4
Some college/ normal	0.632	4.605	3.071	1.534	0.115	22.1	0.077	14.7	57.7	0.038	7.4	9.9
Unknown	0.382	0.390	0.008	0.382	0.004	0.8	0.000	0.0	0.1	0.004	0.8	1.0
All	***0.520***	***0.520***	***0.133***	***0.387***	***0.520***	***100***	***0.133***	***25.5***	***100***	***0.387***	***74.5***	***100***

Source: Author's calculations based on IOF 2014/15.

5 Concluding remarks

I proposed here a detailed decomposition of inequality indices by subpopulations, in which overall inequality can be decomposed into the contribution of the distinct groups making up the population. In the case of additively decomposable inequality indices, these contributions can be further decomposed into their between-group and within-group components. The main approach is based on the RIF of the corresponding inequality index, that extends the classical aggregate decomposition to identify the main drivers of inequality, in a way that is also consistent with the RIF regression-based decomposition of changes in inequality into compositional and structural effects.

The RIF detailed decomposition is straightforward to compute and is shown to verify several appealing properties, such as consistency, path independence, and independence on the level of aggregation among others. However, there are other natural alternatives in the related literature of factor decomposition that can be considered and that do not impose such linearity. I argue that the marginal or Shapley decompositions using the equalizing subpopulation approach are the most relevant tool to attribute the contribution to inequality by subpopulations of additively decomposable indices. They assume that a group with all incomes equal to the population average does not contribute to inequality. I also show that RIF, marginal and Shapley approaches are approximately equal in the case of the Mean Log Deviation, the only index that is additively decomposable and whose decomposition is more genuine and invariant to the path followed to break inequality into between- and within-group components. In other cases, RIF and the other approaches might differ.

The approach was illustrated with an empirical analysis of consumption inequality in Mozambique, a very poor and highly unequal country in sub-Saharan Africa that has shown an increasing trend in recent years. The analysis confirms that the choice of approach is not empirically relevant when it comes to decompose the Mean Log Deviation using the same normalization property. The richest groups, such as people living in Maputo or in other urban areas, with higher educational level, or in the top of the consumption distribution are responsible for the largest shares of inequality and for its increasing trend over time. The value assigned to the contribution of these richest groups to inequality, however, tends to be even higher in the case of the Shapley decomposition compared with RIF when the Theil index is used instead, even if the qualitative results are very similar.

As a final remark, it is interesting to note that one application of this approach is estimating the impact of top incomes (either reported or adjusted) on inequality levels and trends. This can be done for affluent people in general or for specific rich groups, while using aggregate indices of inequality that make the most of the information on the entire distribution, unlike other statistics based only on partial information obtained from incomes at specific quantiles. This helps to analyse one of the topics that are becoming more relevant in the inequality literature (e.g. Atkinson, 2005) using the conventional framework of measuring inequality.

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